24. (3) If all four integers were negative, their product would be positive, and so could not equal one of them. If all four integers were positive, their product would be much greater than any of them (even $1 \times 2 \times 3 \times 4 = 24$). Therefore, the integers must include 0, in which case their product *is* 0. The largest set of four consecutive integers that includes 0 is 0, 1, 2, 3.
25. (7) Since $13^{13} = (13^x)^y = 13^{xy}$, then $xy = 13$. The only positive integers whose product is 13 are 1 and 13. Their average is $\frac{1+13}{2} = 7$.

12-B FRACTIONS AND DECIMALS

Several questions on the SAT involve fractions and/or decimals. In this section we will review all of the important facts on these topics that you need to know for the SAT. Even if you are using a calculator with fraction capabilities, it is essential that you review all of this material thoroughly. (See Chapter 1 for a discussion of calculators that can perform operations with fractions.)

When a whole is *divided* into n equal parts, each part is called *one-nth* of the whole, written as $\frac{1}{n}$. For example, if a pizza is cut (*divided*) into eight equal slices, each slice is one-eighth $\left(\frac{1}{8}\right)$ of the pizza; a day is *divided* into 24 equal hours, so an hour is one-twenty-fourth $\left(\frac{1}{24}\right)$ of a day; and an inch is one-twelfth $\left(\frac{1}{12}\right)$ of a foot.

- If Sam slept for 5 hours, he slept for five-twenty-fourths $\left(\frac{5}{24}\right)$ of a day.
- If Tom bought eight slices of pizza, he bought eight-eighths $\left(\frac{8}{8}\right)$ of a pie.
- If Joe's shelf is 30 inches long, it measures thirty-twelfths $\left(\frac{30}{12}\right)$ of a foot.

Numbers such as $\frac{5}{24}$, $\frac{8}{8}$, and $\frac{30}{12}$, in which one integer is written over a second integer, are called ***fractions***. The center line is the fraction bar. The number above the bar is called the ***numerator***, and the number below the bar is the ***denominator***.

CAUTION: The denominator of a fraction can *never* be 0.

- A fraction such as $\frac{5}{24}$, in which the numerator is less than the denominator, is called a ***proper fraction***. Its value is less than 1.
- A fraction such as $\frac{30}{12}$, in which the numerator is more than the denominator, is called an ***improper fraction***. Its value is greater than 1.
- A fraction such as $\frac{8}{8}$, in which the numerator and denominator are the same, is also an ***improper fraction***, but it is equal to 1.

It is useful to think of the fraction bar as a symbol for division. If three pizzas are divided equally among eight people, each person gets $\frac{3}{8}$ of a pizza. If you actually use your calculator to divide 3 by 8, you get $\frac{3}{8} = 0.375$.

Key Fact B1

Every fraction, proper or improper, can be expressed in decimal form (or as a whole number) by dividing the numerator by the denominator. For example:

$$\frac{3}{10} = 0.3 \quad \frac{3}{4} = 0.75 \quad \frac{5}{8} = 0.625 \quad \frac{3}{16} = 0.1875$$
$$\frac{8}{8} = 1 \quad \frac{11}{8} = 1.375 \quad \frac{48}{16} = 3 \quad \frac{100}{8} = 12.5$$

Note: Any number beginning with a decimal point can be written with a 0 to the left of the decimal point. In fact, some calculators will express $3 \div 8$ as .375, whereas others will print 0.375.

Calculator Shortcut

On the SAT, *never* do long division to convert a fraction to a decimal. Use your calculator.

Unlike the examples above, when most fractions are converted to decimals, the division does not terminate after two, three, or four decimal places; rather it goes on forever with some set of digits repeating itself.

$$\frac{2}{3} = 0.666666\ldots \quad \frac{3}{11} = 0.272727\ldots \quad \frac{5}{12} = 0.416666\ldots \quad \frac{17}{15} = 1.133333\ldots$$

On the SAT, *you do not need to be concerned with this repetition*. On grid-in problems you just enter as much of the number as will fit in the grid; and on multiple-choice questions, all numbers written as decimals terminate.

Although on the SAT you will have occasion to convert fractions to decimals (by dividing), you will not have to convert decimals to fractions.

Comparing Fractions and Decimals

Key Fact B2

To compare two decimals, follow these rules:

- **Whichever number has the greater number to the left of the decimal point is greater: since 11 > 9, 11.001 > 9.896; and since 1 > 0, 1.234 > 0.8. (Recall that, if a decimal has no number to the left of the decimal point, you may assume that a 0 is there, so 1.234 > .8).**
- **If the numbers to the left of the decimal point are equal (or if there are no numbers to the left of the decimal point), proceed as follows:**
  1. **If the numbers do not have the same number of digits to the right of the decimal point, add zeros at the end of the shorter one until the numbers of digits are equal.**
  2. **Now, compare the numbers, *ignoring* the decimal point itself.**

For example, to compare 1.83 and 1.823, add 0 at the end of 1.83, forming 1.830. Now, *thinking of them as whole numbers*, compare the numbers, ignoring the decimal point:

$$1830 > 1823 \Rightarrow 1.830 > 1.823.$$

Key Fact B3

To compare two fractions, use your calculator to convert them to decimals. Then apply KEY FACT B2. This *always* works.

For example, to compare $\frac{1}{3}$ and $\frac{3}{8}$, write

$$\frac{1}{3} = 0.3333\ldots \quad \text{and} \quad \frac{3}{8} = 0.375.$$

Since 0.375 > 0.333, $\frac{3}{8} > \frac{1}{3}$.

CALCULATOR HINT

You can always use your calculator to compare two numbers: fractions, decimals, or integers. By KEY FACT A21, $a > b$ means $a - b$ is positive, and $a < b$ means $a - b$ is negative. Therefore, to compare two numbers, just subtract them. For example,

$$1.83 - 1.823 = .007 \Rightarrow 1.83 > 1.823,$$
$$.2139 - .239 = -.0251 \Rightarrow .2139 < .239,$$
$$\frac{1}{3} - \frac{3}{8} = -\frac{1}{4} \Rightarrow \frac{1}{3} < \frac{3}{8},$$
$$-6 - (-7) = 1 \Rightarrow -6 > -7.$$

Key Fact B4

When comparing fractions, there are three situations in which it is faster *not* to use your calculator to convert fractions to decimals (although, of course, that will work).

1. **The fractions have the same positive denominator. Then the fraction with the larger numerator is greater. Just as \$9 are more than \$7, and 9 books are more than 7 books,**
   9 tenths is more than 7 tenths: $\frac{9}{10} > \frac{7}{10}$.
2. **The fractions have the same numerator. Then, if the denominators are positive, the fraction with the smaller denominator is greater. If you divide a cake into five equal pieces, each piece is larger than a piece you would get if you divided the cake into 10 equal pieces: $\frac{1}{5} > \frac{1}{10}$, and similarly $\frac{3}{5} > \frac{3}{10}$.**
3. **The fractions are so familiar or easy to work with that you already know the answer.**
   For example, $\frac{3}{4} > \frac{1}{5}$ and $\frac{11}{20} > \frac{1}{2}$.

Key Fact B5

KEY FACTS B2, B3, and B4 apply to *positive* decimals and fractions. Clearly, any positive number is greater than any negative number. For negative decimals and fractions, use KEY FACT A25, which states that, if $a > b$, then $-a < -b$.

$$\frac{1}{2} > \frac{1}{5} \Rightarrow -\frac{1}{2} < -\frac{1}{5} \quad \text{and} \quad .83 > .829 \Rightarrow -.83 < -.829$$

Example 1.

Which of the following lists the fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{11}$, and $\frac{13}{20}$ in order from least to greatest?

(A) $\frac{2}{3}, \frac{5}{8}, \frac{7}{11}, \frac{13}{20}$
(B) $\frac{5}{8}, \frac{7}{11}, \frac{13}{20}, \frac{2}{3}$
(C) $\frac{5}{8}, \frac{13}{20}, \frac{7}{11}, \frac{2}{3}$
(D) $\frac{13}{20}, \frac{7}{11}, \frac{5}{8}, \frac{2}{3}$
(E) $\frac{7}{11}, \frac{13}{20}, \frac{2}{3}, \frac{5}{8}$

Solution. On your calculator convert each fraction to a decimal, writing down the first few decimal places:

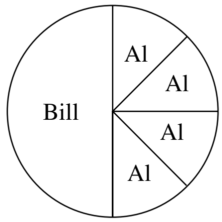
$$\frac{2}{3} = 0.666, \quad \frac{5}{8} = 0.625, \quad \frac{7}{11} = 0.636, \quad \text{and} \quad \frac{13}{20} = 0.65.$$

It is now easy to order the decimals:

$$0.625 < 0.636 < 0.650 < 0.666.$$

The answer is $\mathbf{\frac{5}{8}, \frac{7}{11}, \frac{13}{20}, \frac{2}{3}}$ **(B)**.

Equivalent Fractions



If Bill and Al shared a pizza, and Bill ate $\frac{1}{2}$ and Al ate $\frac{4}{8}$, they had exactly the same amount of the pizza. We express this idea by saying that $\frac{1}{2}$ and $\frac{4}{8}$ are ***equivalent fractions***: that is, they have the exact same value.

Note: If you multiply both the numerator and the denominator of $\frac{1}{2}$ by 4, you get $\frac{4}{8}$; and if you divide both the numerator and the denominator of $\frac{4}{8}$ by 4, you get $\frac{1}{2}$. This illustrates the next KEY FACT.

Key Fact B6

Two fractions are equivalent if multiplying or dividing both the numerator and the denominator of the first fraction *by the same number* gives the second fraction.

Consider the following two cases.

1. Are $\frac{3}{8}$ and $\frac{45}{120}$ equivalent? There is a number that, when multiplied by 3 gives 45, and there is a number that, when multiplied by 8, gives 120. By KEY FACT B6, if these numbers are the same, the fractions are equivalent. They *are* the same number: $3 \times 15 = 45$ and $8 \times 15 = 120$.
2. Are $\frac{2}{3}$ and $\frac{28}{45}$ equivalent? Since $2 \times 14 = 28$, but $3 \times 14 \neq 45$, the fractions are *not* equivalent. Alternatively, $28 \div 14 = 2$, but $45 \div 14 \neq 3$.

Calculator Shortcut

To determine whether two fractions are equivalent, convert them to decimals by dividing. For the fractions to be equivalent, the two quotients must be the same.

Example 2

Which of the following is NOT equivalent to $\frac{15}{24}$?

(A) $\frac{45}{72}$ (B) $\frac{60}{96}$ (C) $\frac{180}{288}$ (D) $\frac{5}{8}$ (E) $\frac{3}{5}$

Solution. Since $\frac{15}{24} = 0.625$, just check each choice until you find the one that is NOT equal to 0.625. Each of $\frac{45}{72}$, $\frac{60}{96}$, $\frac{180}{288}$, and $\frac{5}{8}$ is equal to 0.625. Only $\mathbf{\frac{3}{5}}$ **(E)** does not equal 0.625 $\left(\frac{3}{5} = 0.6\right)$.

A fraction is in ***lowest terms*** if no positive integer greater than 1 is a factor of both the numerator and the denominator. For example, $\frac{9}{20}$ is in lowest terms, since no integer greater than 1 is a factor of both 9 and 20; but $\frac{9}{24}$ is not in lowest terms, since 3 is a factor of both 9 and 24.

Key Fact B7

Every fraction can be *reduced* to lowest terms by dividing the numerator and the denominator by their greatest common factor (GCF). If the GCF is 1, the fraction is already in lowest terms.

Calculator Shortcut

Calculators that have fraction capability either reduce automatically or have a key to reduce. On a regular calculator, see whether some prime can divide evenly into both the numerator and the denominator. On the SAT, if none of 2, 3, 5, 7, or 11 works, the fraction cannot be reduced.

Helpful Hints

Keep these two facts in mind:

1. On grid-in problems you should never reduce a fraction, such as $\frac{9}{24}$, that fits in a grid; just enter it: $\boxed{9\,/\,2\,4}$. If a fraction can't fit, such as $\frac{36}{63}$, don't try to reduce it; use your calculator to convert it into a decimal: $\frac{36}{63} = .571428...$, and enter $\boxed{.\,5\,7\,1}$.
   You should reduce a fraction only if you are positive you can do so in *1 or 2 seconds* (faster than you could divide on your calculator), *without making a mistake*. Examples of easy reductions are $\frac{100}{200} = \frac{1}{2}$, $\frac{20}{30} = \frac{2}{3}$, $\frac{2}{120} = \frac{1}{60}$.
2. On multiple-choice questions, on the other hand, the fraction choices are almost always in lowest terms, so you may have to reduce your answer to see which choice is correct. If you can't do this easily in your head, use your calculator either to reduce the fraction or to convert it to a decimal, and then use your calculator again to check the five choices.

Example 3.

For any positive integer n, $n!$ means the product of all the integers from 1 to n. What is the value of $\frac{6!}{8!}$?

(A) $\frac{1}{56}$ (B) $\frac{1}{48}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{3}{4}$

Solution. Assume that you don't see the easy way to do this. On your calculator quickly multiply (or use the ! key if you have one):

$$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720,$$
$$8! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 = 40{,}320.$$

You are now faced with reducing $\frac{720}{40{,}320}$. Don't do it.

Use your calculator to divide: $\frac{720}{40{,}320} = 0.0178...$. Now test the choices, starting with C: $\frac{1}{8} = 0.125$, which is too large. Eliminate C as well as D and E, which are even larger, and try A or B. In fact, $\frac{1}{56} = 0.0178....$ Choice **A** is correct. Here's the easy solution:

$$\frac{6!}{8!} = \frac{\overset{1}{\cancel{6\times5\times4\times3\times2\times1}}}{8\times7\times\underset{1}{\cancel{6\times5\times4\times3\times2\times1}}} = \frac{1}{8\times7} = \mathbf{\frac{1}{56}}.$$

This solution takes only a few seconds, but the calculator solution is simple enough and can surely be done in less than a minute.

Arithmetic Operations with Decimals

Calculator Shortcut

On the SAT, *all* decimal arithmetic (including whole numbers) that you can't easily do in your head should be done on your calculator.

This shortcut saves time and avoids careless errors. If you know that $12 \times 12 = 144$ and that $1.2 \times 1.2 = 1.44$, fine; but if you're not sure, use your calculator rather than your pencil. You should even use your calculator to multiply 0.2×0.2 if there's any chance that you would get 0.4 instead of 0.04 as the answer.

You should *not* have to use your calculator to multiply or divide any decimal number by a power of 10, because multiplying and dividing by 10 or 100 or 1000 is a calcuation you should be able to do easily in your head.

Helpful Hint

Any whole number can be treated as a decimal: 7 = 7.0.

Key Fact B8

To multiply any decimal or whole number by a power of 10, move the decimal point as many places to the right as there are 0's in the power of 10, filling in with 0's if necessary.

$$1.35 \times 10 = 1\underbrace{3}_{1}.5 \qquad 1.35 \times 100 = 1\underbrace{35}_{2}$$
$$1.35 \times 1000 = 1\underbrace{350}_{3}$$
$$23 \times 10 = 23\underbrace{0}_{1} \qquad 23 \times 100 = 23\underbrace{00}_{2}$$
$$23 \times 1{,}000{,}000 = 23{,}\underbrace{000{,}000}_{6}$$

Key Fact B9

To divide any decimal or whole number by a power of 10, move the decimal point as many places to the left as there are 0's in the power of 10, filling in with 0's if necessary.

$$67.8 \div 10 = 6.\underbrace{7}_{1}8 \qquad 67.8 \div 100 = 0.\underbrace{67}_{2}8$$
$$67.8 \div 1000 = 0.\underbrace{067}_{3}8$$
$$14 \div 10 = 1.\underbrace{4}_{1} \qquad 14 \div 100 = 0.\underbrace{14}_{2}$$
$$14 \div 1{,}000{,}000 = 0.\underbrace{000014}_{6}$$

On the SAT, you *never* have to round off decimal answers. On grid-ins just enter the number, putting in as many digits after the decimal point as fit. For example, enter 3.125 as $\boxed{3}\boxed{.}\boxed{1}\boxed{2}$ and .1488 as $\boxed{.}\boxed{1}\boxed{4}\boxed{8}$. However, you do have to know how to round off, because *occasionally* there is a question about that procedure.

Key Fact B10

To *round off* a decimal number to any place, follow these rules, which are fully explained with examples in the table below.

- **Keep all of the digits to the left of the specified place.**
- **In that place, keep the digit if the next digit is < 5, and increase that digit by 1 if the next digit is ≥ 5. (*Note:* 9 increased by 1 is 10: put down the 0 and carry the 1.)**
- **If there are still digits to the left of the decimal point, change them to 0's and eliminate the decimal point and everything that follows it.**
- **If you are at or beyond the decimal point, stop: don't write any more digits.**

For example, here is how to round off 3815.296 to any place.

Round to the Nearest:	Procedure	Answer
thousand	The digit in the thousands place is 3; since the next digit (8) is ≥ 5, increase the 3 to a 4; fill in the 3 places to the left of the decimal point with 0's.	4000
hundred	The digit in the hundreds place is 8; keep everything to the left of it, and keep the 8 since the next digit (1) is < 5; fill in 0's to the left of the decimal point.	3800
ten	The digit in the tens place is 1; keep everything to the left of it, and increase the 1 to a 2 since the next digit (5) is ≥ 5; fill in 0's to the left of the decimal point.	3820
one	The digit in the ones place is 5; keep everything to the left of it, and keep the 5 since the next digit (2) is < 5; there are no more places to the left of the decimal point, so stop.	3815
tenth	The digit in the tenths place is 2; keep everything to the left of it, and increase the 2 to a 3 since the next digit (9) is ≥ 5; you are beyond the decimal point, so stop.	3815.3
hundredth	The digit in the hundredths place is 9; keep everything to the left of it, and, since the next digit (6) is ≥ 5, increase the 9 to a 10; put down the 0 and carry a 1 into the tenths place: 0.29 becomes 0.30; since you are beyond the decimal point, stop.	3815.30

Example 4.

When 423,890 is rounded off to the nearest thousand, how many digits will be changed?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Solution. When 42$\underline{3,89}$0 is rounded off to the nearest thousand, **3** digits are changed: 42$\underline{4,00}$0 **(D)**.

Arithmetic Operations with Fractions

Key Fact B11

To multiply two fractions, multiply their numerators and multiply their denominators.

$$\frac{3}{5}\times\frac{4}{7}=\frac{3\times 4}{5\times 7}=\frac{12}{35}.$$

Key Fact B12

To multiply a fraction by any other number, write that number as a fraction whose denominator is 1.

$$\frac{3}{5}\times 7=\frac{3}{5}\times\frac{7}{1}=\frac{21}{5} \qquad \frac{3}{4}\times\pi=\frac{3}{4}\times\frac{\pi}{1}=\frac{3\pi}{4}$$

Tactic B1

Before multiplying fractions, reduce. You may reduce by dividing any numerator and any denominator by a common factor.

Example 5.

Express the product $\frac{3}{4}\times\frac{8}{9}\times\frac{15}{16}$ in lowest terms.

Solution. If you just multiply the numerators and denominators (with a calculator, of course), you get $\frac{360}{576}$, which is a nuisance to reduce. Also, dividing on your calculator won't help, since your answer is supposed to be a fraction in lowest terms. It is better to use TACTIC B1 and reduce first:

$$\frac{\overset{1}{\cancel{3}}}{4}\times\frac{\overset{1}{\cancel{8}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{9}}}\times\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}}=\frac{1\times1\times5}{4\times1\times2}=\frac{\mathbf{5}}{\mathbf{8}}.$$

Tactic B2

When a problem requires you to find a fraction *of* a number, multiply.

Example 6.

If $\frac{4}{7}$ of the 350 sophomores at Adams High School are girls, and $\frac{7}{8}$ of the girls play on a team, how many sophomore girls do NOT play on a team?

Solution. There are $\frac{4}{7}\times 350 = 200$ sophomore girls. Of these, $\frac{7}{8}\times 200 = 175$ play on a team. Then, $200 - 175 =$ **25** do not play on a team.

How should you multiply $\frac{4}{7}\times 350$? If you can do this mentally, you should:

$$\frac{4}{\underset{1}{\cancel{7}}}\times\overset{50}{\cancel{350}}=200.$$

The next step, however, requires you to multiply $\frac{7}{8}$ by 200, and more likely than not you don't *immediately* see that 200 divided by 8 is 25 or that 7 times 25 equals 175:

$$\frac{7}{\cancel{8}_{1}} \times \cancel{200}^{25} = 175.$$

For any step that you can't do instantly, you should use your calculator:

$$(4 \div 7) \times 350 \times (7 \div 8) = 175.$$

CALCULATOR HINT

If you are going to use your calculator on a problem, don't bother reducing anything. Given the choice of multiplying $\frac{48}{128} \times 80$ or $\frac{3}{8} \times 80$, you would prefer the second option, but for *your calculator* the first one is just as easy.

The ***reciprocal*** of any nonzero number, *x,* is the number $\frac{1}{x}$. The reciprocal of the fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$.

Key Fact B13

To divide any number by a fraction, multiply the number by the reciprocal of the fraction.

$$20 \div \frac{2}{3} = \frac{\cancel{20}^{10}}{1} \times \frac{3}{\cancel{2}_{1}} = 30 \qquad \frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

$$\sqrt{2} \div \frac{2}{3} = \frac{\sqrt{2}}{1} \times \frac{3}{2} = \frac{3\sqrt{2}}{2}$$

Helpful Hint

Even if you have a calculator with fraction capability, be sure to review the rules in KEY FACTS B11–B15. Some calculations are easier without a calculator.

Example 7.

In the meat department of a supermarket, 100 pounds of chopped meat was divided into packages, each of which weighed $\frac{4}{7}$ pound. How many packages were there?

Solution. $100 \div \frac{4}{7} = \frac{\cancel{100}^{25}}{1} \times \frac{7}{\cancel{4}_{1}} = \mathbf{175}$.

Key Fact B14

To add or subtract fractions with the same denominator, add or subtract the numerators and keep the denominator.

$$\frac{4}{9} + \frac{1}{9} = \frac{5}{9} \quad \text{and} \quad \frac{4}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3}.$$

To add or subtract fractions with different denominators, first rewrite the fractions as equivalent fractions with the same denominator.

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}.$$

Note: The *easiest* denominator to find is the product of the denominators (6 × 4 = 24, in this example), but the *best* denominator to use is the *least common denominator*, which is the least common multiple (LCM) of the denominators (12 in this case). Using the least common denominator minimizes the amount of reducing that is necessary to express the answer in lowest terms.

Key Fact B15

If $\frac{a}{b}$ is the fraction of the whole that satisfies some property, then $1 - \frac{a}{b}$ is the fraction of the whole that does *not* satisfy it.

Example 8.

In a jar, $\frac{1}{2}$ of the marbles are red, $\frac{1}{4}$ are white, and $\frac{1}{5}$ are blue. What fraction of the marbles are neither red, white, nor blue?

Solution. The red, white, and blue marbles constitute

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10}{20} + \frac{5}{20} + \frac{4}{20} = \frac{19}{20}$$

of the total, so

$$1 - \frac{19}{20} = \frac{20}{20} - \frac{19}{20} = \mathbf{\frac{1}{20}}$$

of the marbles are neither red, white, nor blue.

Example 9.

Ali ate $\frac{1}{3}$ of a cake and Jason ate $\frac{1}{4}$ of it. What fraction of the cake was still uneaten?

Example 10.

Ali ate $\frac{1}{3}$ of a cake and Jason ate $\frac{1}{4}$ of what was left. What fraction of the cake was still uneaten?

CAUTION: Be sure to read questions carefully. In Example 9, Jason ate $\frac{1}{4}$ of the cake. In Example 10, however, he ate only $\frac{1}{4}$ of the $\frac{2}{3}$ that was left after Ali had her piece. He ate

$$\frac{1}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{3} = \frac{1}{6}$$

of the cake.

Solution 9. $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$ of the cake was eaten, and $1 - \frac{7}{12} = \mathbf{\frac{5}{12}}$ was uneaten.

Solution 10. $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$ of the cake was eaten, and the other $\mathbf{\frac{1}{2}}$ was uneaten.

Arithmetic Operations with Mixed Numbers

A ***mixed number*** is a number, such as $3\frac{1}{2}$, that consists of an integer followed by a fraction. The mixed number is an abbreviation for the *sum* of the integer and the fraction; so $3\frac{1}{2}$ is an abbreviation for $3 + \frac{1}{2}$.

Every mixed number can be written as an improper fraction, and every improper fraction can be written as a mixed number:

$$3\frac{1}{2} = 3 + \frac{1}{2} = \frac{3}{1} + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

and $$\frac{7}{2} = \frac{6}{2} + \frac{1}{2} = 3 + \frac{1}{2} = 3\frac{1}{2}.$$

Key Fact B16

To write a mixed number as an improper fraction, or an improper fraction as a mixed number, follow these rules:

1. **To write a mixed number $\left(3\frac{1}{2}\right)$ as an improper fraction, multiply the whole number (3) by the denominator (2), add the numerator (1), and write the sum over the denominator (2): $\frac{3 \times 2 + 1}{2} = \frac{7}{2}$.**
2. **To write an improper fraction $\left(\frac{7}{2}\right)$ as a mixed number, divide the numerator by the denominator; the quotient (3) is the whole number. Place the remainder (1) over the denominator to form the fractional part $\left(\frac{1}{2}\right)$: $3\frac{1}{2}$.**

CAUTION: You can *never* grid in a mixed number. You must change it to an improper fraction or a decimal.

Key Fact B17

To add mixed numbers, add the integers and also add the fractions.

$$5\frac{1}{4} + 3\frac{2}{3} = (5 + 3) + \left(\frac{1}{4} + \frac{2}{3}\right) = 8 + \left(\frac{3}{12} + \frac{8}{12}\right) = 8 + \frac{11}{12} = 8\frac{11}{12}.$$

$$5\frac{3}{4} + 3\frac{2}{3} = (5 + 3) + \left(\frac{3}{4} + \frac{2}{3}\right) = 8 + \left(\frac{9}{12} + \frac{8}{12}\right) = 8 + \frac{17}{12} = 8 + 1\frac{5}{12} = 8 + 1 + \frac{5}{12} = 9\frac{5}{12}$$

Key Fact B18

To subtract mixed numbers, subtract the integers and also subtract the fractions. If, however, the fraction in the second number is greater than the fraction in the first number, you first have to borrow 1 from the integer part.

For example, since $\frac{2}{3} > \frac{1}{4}$, you can't subtract $5\frac{1}{4} - 3\frac{2}{3}$ until you borrow 1 from the 5:

$$5\frac{1}{4} = 5 + \frac{1}{4} = (4 + 1) + \frac{1}{4} = 4 + \left(1 + \frac{1}{4}\right) = 4 + \frac{5}{4}.$$

Now, you have

$$5\frac{1}{4} - 3\frac{2}{3} = 4\frac{5}{4} - 3\frac{2}{3} = (4 - 3) + \left(\frac{5}{4} - \frac{2}{3}\right) = 1 + \left(\frac{15}{12} - \frac{8}{12}\right) = 1\frac{7}{12}.$$

Key Fact B19

To multiply or divide mixed numbers, change them to improper fractions.

$$1\frac{2}{3} \times 3\frac{1}{4} = \frac{5}{3} \times \frac{13}{4} = \frac{65}{12} = 5\frac{5}{12}.$$

CAUTION: Be aware that $3\left(5\frac{1}{2}\right)$ is *not* $15\frac{1}{2}$; rather:

$$3\left(5\frac{1}{2}\right) = 3\left(5 + \frac{1}{2}\right) = 15 + \frac{3}{2} = 15 + 1\frac{1}{2} = 16\frac{1}{2}.$$

Calculator Shortcut

All arithmetic operations on mixed numbers can be done directly on calculators with fraction capability; there is no need to change the mixed numbers to improper fractions or to borrow.

Complex Fractions

A ***complex fraction*** is a fraction, such as $\dfrac{1+\frac{1}{6}}{2-\frac{3}{4}}$, that has one or more fractions in its numerator or denominator or both.

Key Fact B20

There are two ways to simplify a complex fraction:

1. **Multiply *every* term in the numerator and denominator by the least common multiple of all the denominators that appear in the fraction.**
2. **Simplify the numerator and the denominator, and divide.**

To simplify $\dfrac{1+\frac{1}{6}}{2-\frac{3}{4}}$, multiply each term by 12, the LCM of 6 and 4:

$$\frac{12(1)+\overset{2}{\cancel{12}}\left(\frac{1}{\cancel{6}_1}\right)}{12(2)-\overset{3}{\cancel{12}}\left(\frac{3}{\cancel{4}_1}\right)}=\frac{12+2}{24-9}=\frac{14}{15},$$

or write

$$\frac{1+\frac{1}{6}}{2-\frac{3}{4}}=\frac{\frac{7}{6}}{\frac{5}{4}}=\frac{7}{\underset{3}{\cancel{6}}}\times\frac{\overset{2}{\cancel{4}}}{5}=\frac{14}{15}.$$

Calculator Shortcut

Remember that, on the SAT, if you ever get stuck on a fraction problem, you can always convert the fractions to decimals and do all the work on your calculator.

Exercises on Fractions and Decimals

Multiple-Choice Questions

1. A French class has 12 boys and 18 girls. Boys are what fraction of the class?

   (A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

2. For how many integers, a, between 30 and 40 is it true that $\frac{5}{a}$, $\frac{8}{a}$, and $\frac{13}{a}$ are all in lowest terms?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. What is the value of the product $\frac{5}{5}\times\frac{5}{10}\times\frac{5}{15}\times\frac{5}{20}\times\frac{5}{25}$?

   (A) $\frac{1}{120}$ (B) $\frac{1}{60}$ (C) $\frac{1}{30}$ (D) $\frac{5}{30}$ (E) $\frac{1}{2}$

4. Billy won some goldfish at the state fair. During the first week, $\frac{1}{5}$ of them died; and during the second week, $\frac{3}{8}$ of those still alive at the end of the first week died. What fraction of the original goldfish were still alive after 2 weeks?

   (A) $\frac{3}{10}$ (B) $\frac{17}{40}$ (C) $\frac{1}{2}$ (D) $\frac{23}{40}$ (E) $\frac{7}{10}$

5. $\frac{1}{4}$ is the average (arithmetic mean) of $\frac{1}{5}$ and what number?

   (A) $\frac{1}{20}$ (B) $\frac{3}{10}$ (C) $\frac{1}{3}$ (D) $\frac{9}{20}$ (E) $\frac{9}{40}$

6. If $\frac{3}{11}$ of a number is 22, what is $\frac{6}{11}$ of that number?

   (A) 6 (B) 11 (C) 12 (D) 33 (E) 44

7. What fractional part of a week is 98 hours?

   (A) $\frac{7}{24}$ (B) $\frac{24}{98}$ (C) $\frac{1}{2}$ (D) $\frac{4}{7}$ (E) $\frac{7}{12}$

8. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?

   (A) 7 (B) 8 (C) 15 (D) $\frac{7}{225}$ (E) $\frac{225}{7}$

9. Which of the following is less than $\frac{5}{9}$?

   (A) $\frac{5}{8}$ (B) $\frac{21}{36}$ (C) $\frac{25}{45}$ (D) $\frac{55}{100}$ (E) .565

10. Which of the following is (are) greater than x when $x = \frac{9}{11}$?

I. $\frac{1}{x}$

II. $\frac{x+1}{x}$

III. $\frac{x+1}{x-1}$

(A) I only (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

11. Which of the following statements is true?

(A) $\frac{3}{8} < \frac{4}{11} < \frac{5}{13}$ (B) $\frac{4}{11} < \frac{3}{8} < \frac{5}{13}$

(C) $\frac{5}{13} < \frac{4}{11} < \frac{3}{8}$ (D) $\frac{4}{11} < \frac{5}{13} < \frac{3}{8}$

(E) $\frac{3}{8} < \frac{5}{13} < \frac{4}{11}$

12. If $a = 0.99$, which of the following is (are) less than a?

I. $\sqrt{a}$

II. a^2

III. $\frac{1}{a}$

(A) None (B) I only (C) II only (D) III only
(E) II and III only

13. Let a, b, c, and d be the result of rounding off 7382.196 to the nearest thousand, hundred, ten, and one, respectively. Which of the following statements is true?

(A) $d < c < b < a$ (B) $d < c < a < b$
(C) $a < d < c < b$ (D) $c < d < b < a$
(E) $a < c < d < b$

14. For what value of x does $\frac{(34.56)(7.89)}{x} = (0.3456)(78.9)$?

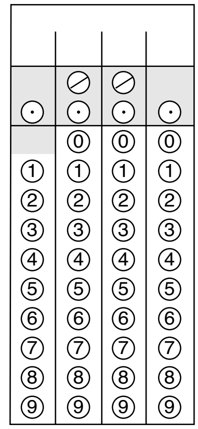
(A) 0.001 (B) 0.01 (C) 0.1 (D) 10 (E) 100

15. For the final step in a calculation, Paul accidentally divided by 1000 instead of multiplying by 1000. What should he do to his answer to correct it?

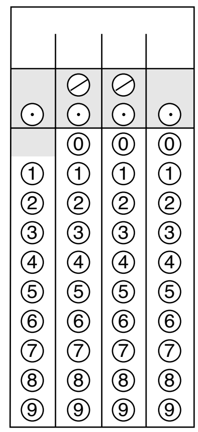
(A) Multiply it by 1000.
(B) Multiply it by 100,000.
(C) Multiply it by 1,000,000.
(D) Square it.
(E) Double it.

Grid-in Questions

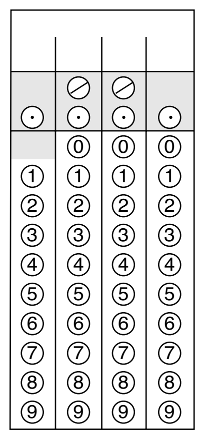
16. One day at Central High School, $\frac{1}{12}$ of the students were absent, and $\frac{1}{5}$ of those present went on a field trip. If the number of students staying in school was 704, how many students are enrolled at Central High?



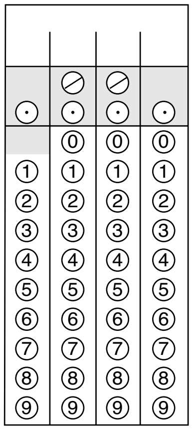
17. What is a possible value of x if $\frac{3}{5} < \frac{1}{x} < \frac{7}{9}$?



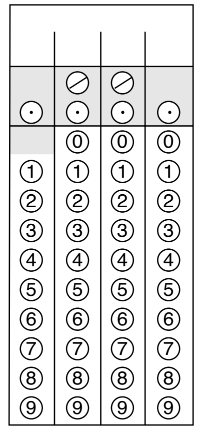
18. What is the value of $\dfrac{\frac{7}{9} \times \frac{7}{9}}{\frac{7}{9} + \frac{7}{9} + \frac{7}{9}}$?



19. If $7a = 3$ and $3b = 7$, what is the value of $\frac{a}{b}$?



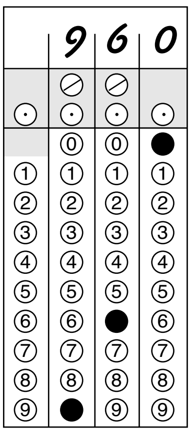
20. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and C is the set consisting of all the fractions whose numerators are in A and whose denominators are in B, what is the product of all of the numbers in C?



Answer Key

1.	**A**	4.	**C**	7.	**E**	10.	**B**	13.	**E**
2.	**C**	5.	**B**	8.	**A**	11.	**B**	14.	**D**
3.	**A**	6.	**E**	9.	**D**	12.	**C**	15.	**C**

16.



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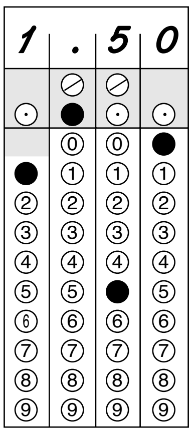
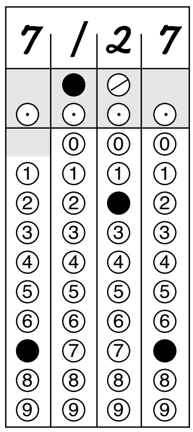


$1.28 < x < 1.67$

18.



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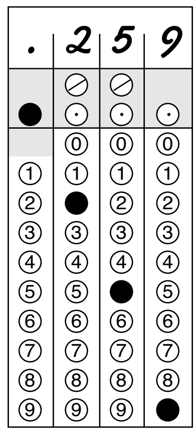

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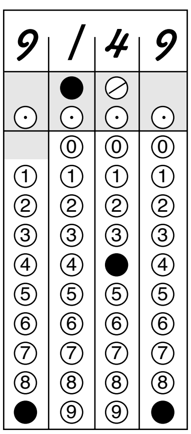

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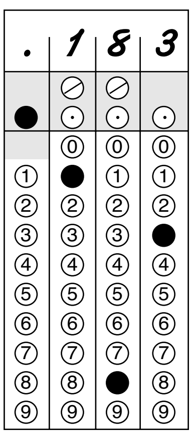

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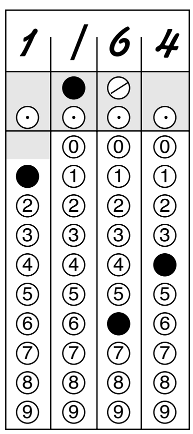

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